

# Updated constraints on new physics in rare charm decays

Svjetlana Fajfer, Nejc Košnik and Saša Prelovšek

*Department of Physics, University of Ljubljana,  
Jadranska 19, 1000 Ljubljana, Slovenia*

and

*J. Stefan Institute, Jamova 39, P. O. Box 300, 1001 Ljubljana*

## ABSTRACT

Motivated by recent experimental results on charm physics we investigate implications of the updated constraints of new physics in rare charm meson decays. We first reconsider effects of the MSSM in  $c \rightarrow u\gamma$  constrained by the recent experimental evidence on  $\Delta m_D$  and find, that due to the dominance of long distance physics,  $D \rightarrow V\gamma$  decay rates cannot be modified by MSSM contributions. Then we consider effects of the extra heavy up vector-like quark models on the decay spectrum of  $D^+ \rightarrow \pi^+\ell^+\ell^-$  and  $D_s^+ \rightarrow K^+\ell^+\ell^-$  decays. We find a possibility for the tiny increase of the differential decay rate in the region of large dilepton mass. The  $R$ -parity violating supersymmetric model can also modify short distance dynamics in  $c \rightarrow u\ell^+\ell^-$  decays. We constrain relevant parameters using current upper bound on the  $D^+ \rightarrow \pi^+\ell^+\ell^-$  decay rate and investigate impact of that constraint on the  $D_s^+ \rightarrow K^+\ell^+\ell^-$  differential decay dilepton distribution. Present bounds still allow small modification of the standard model differential decay rate distribution.

## I. INTRODUCTION

Recently the evidence for  $D^0 - \bar{D}^0$  oscillations has been reported by Belle and BaBar collaborations [1, 2]. Combining the measured quantities [3] indicates nonzero  $\Delta\Gamma$  as well as nonzero  $\Delta m_D$

$$x = \Delta m_D / \Gamma_D = 0.0087 \pm 0.003 . \quad (1)$$

These results immediately stimulated many studies (see eg. [4, 5, 6, 7, 8, 9]). The obtained results for the relevant parameters describing  $D^0 - \bar{D}^0$  mixing are not in favor of new physics effects. However, they give additional constraints on physics beyond the standard model (SM) as already given in papers [5, 6]. On the other hand, the study of rare  $D$  meson decays is not considered to be very informative in current searches of physics beyond the standard model [10, 11, 12, 13, 14, 15, 16, 17], as it is expected from “b” physics. Namely, most of the charm meson processes, where the flavor changing neutral currents (FCNC) effects might be present like  $c \rightarrow u$  and  $c\bar{u} \leftrightarrow \bar{c}u$  transitions, are dominated by the standard model long-distance contributions [10, 11, 12, 13, 14, 15, 16, 17, 18].

Due to the GIM mechanism and smallness of the down-type quark masses the radiative  $c \rightarrow u\gamma$  decay rate is strongly suppressed at the leading order in the SM [10, 14]. The QCD effects enhance it up to the order of  $10^{-8}$  [19]. New bounds on the mass insertion parameters within minimal supersymmetric SM (MSSM) [4, 5] are derived using the  $D^0 - \bar{D}^0$  oscillations. We include into consideration the possible effect of MSSM with non-universal soft breaking terms on  $c \rightarrow u\gamma$  along the lines of [20, 21]. Although this approach leads to the enhancement of the SM value by a factor 10, it is too small to give any observable effects in  $D \rightarrow V\gamma$  decays ( $V$  is a light vector meson). The dominating long-distance (LD) contributions in the  $D \rightarrow V\gamma$  decays give the branching ratios of the order  $\text{Br} \sim 10^{-6}$  [10, 14], which makes the search for new physics effects impossible.

Another possibility to search for the effects of new physics in the charm sector is offered in the studies of  $D \rightarrow X\ell^+\ell^-$  decays which might be result of the  $c \rightarrow u\ell^+\ell^-$  FCNC transition [7, 11, 12, 15, 16, 18]. Here  $X$  can be light vector meson  $V$  or pseudoscalar meson  $P$ . Within SM inclusion of renormalization group improved QCD corrections for the inclusive  $c \rightarrow u\ell^+\ell^-$  gave an additional significant suppression leading to the rates  $\Gamma(c \rightarrow ue^+e^-)/\Gamma_{D^0} = 2.4 \times 10^{-10}$  and  $\Gamma(c \rightarrow u\mu^+\mu^-)/\Gamma_{D^0} = 0.5 \times 10^{-10}$  [22]. These transitions are largely driven by a virtual photon at low dilepton mass  $m_{\ell\ell} \equiv \sqrt{(p_+ + p_-)^2}$ , while the total rate for  $D \rightarrow X\ell^+\ell^-$  is dominated by the LD resonant contributions at dilepton masses  $m_{\ell\ell} = m_\rho, m_\omega, m_\phi$  [11, 16].

New physics could possibly modify the dilepton mass distribution below  $\rho$  or distribution above

$\phi$  resonance. In the case of  $D \rightarrow \pi \ell^+ \ell^-$  there is a broad kinematical region of dilepton mass above  $\phi$  resonance which presents an unique possibility to study  $c \rightarrow u \ell^+ \ell^-$  at high  $m_{\ell\ell}$  [16].

The leading contribution to  $c \rightarrow u \ell^+ \ell^-$  in general MSSM with the conserved  $R$ -parity comes from one-loop diagram with gluino and squarks in the loop [11, 16, 23]. It proceeds via virtual photon and enhances the  $c \rightarrow u \ell^+ \ell^-$  spectrum at small  $m_{\ell\ell}$ . We find that bounds on the mass insertion parameters make the abovementioned enhancement in  $D$  rare decay [11, 12] to be negligible.

Some models of new physics contain an extra up-like heavy quark singlet [24] inducing the FCNCs at tree level for the up-quark sector [13, 25, 26, 27, 28], while the neutral current for the down-like quarks is the same as in the SM. The stringest bound on these models comes from the recent bound on  $\Delta m$  in the  $D^0 - \bar{D}^0$  transition as given in [1, 2]. In our calculation, we analyze how these bounds on the FCNC vertex  $cuZ$  affect the  $D \rightarrow P \ell^+ \ell^-$  decays. A particular version of the model with tree-level up-quark FCNC transitions is the Littlest Higgs model [29]. In this case the magnitude of the relevant  $c \rightarrow uZ$  coupling is even further constrained by the large scale  $f \geq \mathcal{O}(1 \text{ TeV})$  using the precision electroweak data. The smallness implies that the effect of this particular model on  $c \rightarrow u \ell^+ \ell^-$  decay and relevant rare  $D$  decays is insignificant [13].

Among discussed models of new physics the supersymmetric extension of the SM including the  $R$ -parity violation is still not constrained as other new physics models. As noticed by [11, 22] one can test some combinations of the  $R$ -parity violating contributions in  $D^+ \rightarrow \pi^+ \ell^+ \ell^-$  decays. We place new constraints on the relevant parameters and search for the effects of new physics in the  $D_s^+ \rightarrow K^+ \ell^+ \ell^-$  decays which might be interesting for the experimental studies.

There are intensive experimental efforts by CLEO [30, 31] and FERMILAB [32, 33] collaborations to improve the upper limits on the rates for  $D \rightarrow X \ell^+ \ell^-$  decays. Two events in the channel  $D^+ \rightarrow \pi^+ e^+ e^-$  with  $m_{ee}$  close to  $m_\phi$  have already been observed by CLEO [30]. The other rare  $D$  meson decays are not so easily accessible by experimental searches, but with the plans to make more experimental studies in rare charm decays at CLEO-c, Tevatron and at charm physics sections at present  $B$ -factories and in the future at LHC-b facilities makes the study of rare  $D$  decays more attractive.

In order to compare effects of new physics and the standard model we have to determine size of the LD contributions. As in the case of  $D^+ \rightarrow \pi^+ e^+ e^-$  decay we use experimental data on the  $D_s$  nonleptonic decays accompanied by the vector meson dominance as we did in [13].

The paper is organized as follows. In Section 2 we consider the impact of new charm mixing bounds on the  $c \rightarrow u \gamma$  decay. In Sec. 3 we study how new physics affects  $c \rightarrow u \ell^+ \ell^-$  and  $D \rightarrow P \ell^+ \ell^-$  decays. We present framework for calculating SD effects as well as the details of LD

calculations. In Sec. 4 we discuss our results and in Sec. 5 we make a short summary.

## II. $c \rightarrow u\gamma$ DECAY

Given the recent observation of  $D^0 - \bar{D}^0$  mixing, we re-evaluate the possible effect of MSSM on  $c \rightarrow u\gamma$ . Since the model with universal soft-breaking terms is known to have negligible effect [21], we consider the model with non-universal soft breaking terms. We consider only the gluino exchange diagrams through  $(\delta_{12}^u)_{LR,RL}$  mass insertions, since the remaining SUSY contributions can not have sizable effect [20, 21]. The maximal value of  $(\delta_{12}^u)_{LR,RL}$  insertion has been constrained by saturating  $x = \Delta m_D/\Gamma = (4.8 \pm 2.8) \times 10^{-3}$  with the gluino exchange in [5]. The results corresponding to the measured  $x = (8.7 \pm 3) \times 10^{-3}$  [3, 9] are shown in second column of Table I. Another constraint is obtained by requiring the minima of MSSM scalar potential do not break electric charge or color and that they are bounded from above  $(\delta_{12}^u)_{LR,RL} \leq \sqrt{3}m_c/m_{\tilde{q}}$  [34], with values given in third column of Table I. The second constraint is obviously stronger for  $m_{\tilde{q}} \geq 350$  GeV, while  $\Delta m_D$  gives more stringent constraint for lighter squarks. Using  $(\delta_{12}^u)_{LR,RL} \leq \sqrt{3}m_c/m_{\tilde{q}}$ ,  $m_{\tilde{q}} = m_{\tilde{g}} = 350$  GeV,  $m_c = 1.25$  GeV and expressions from [21] we get the upper bound

$$\Gamma(c \rightarrow u\gamma)/\Gamma_{D^0} \leq 8 \times 10^{-7}, \quad (2)$$

which is one order of magnitude larger than the standard model prediction  $\Gamma(c \rightarrow u\gamma)/\Gamma_{D^0} = 2.5 \times 10^{-8}$  [19].

$m_{\tilde{q}} = m_{\tilde{g}}$	$ (\delta_{12}^u)_{LR,RL} $ from $\Delta m_D$	$ (\delta_{12}^u)_{LR,RL} $ from stability bound
350 GeV	0.007	0.006
500 GeV	0.01	0.004
1000 GeV	0.02	0.002

Table I: Upper bounds on mass insertions  $|(\delta_{12}^u)_{LR,RL}|$  from measured  $\Delta m_D$  and stability bound [34].

However, this possible SUSY enhancement by factor 10 would not affect the rate of the  $D \rightarrow V\gamma$  decays, which are completely dominated by LD contributions with  $\text{Br} \sim 10^{-6}$  [10, 11, 12, 14, 18]. The only window for probing the  $c \rightarrow u\gamma$  enhancement remains the  $B_c \rightarrow B_u^*\gamma$  decay, where LD contributions are strongly suppressed [35].

### III. $c \rightarrow u\ell^+\ell^-$ AND $D \rightarrow P\ell^+\ell^-$ DECAYS

#### A. SD Effects

The  $c \rightarrow u\ell^+\ell^-$  transition is driven by the low-energy effective Lagrangian

$$\mathcal{L}_{\text{eff}}^{\text{SD}} = \frac{G_F}{\sqrt{2}} V_{cb}^* V_{ub} \sum_{i=7,9,10} C_i Q_i, \quad (3)$$

given in terms of four-quark operators

$$Q_7 = \frac{e}{8\pi^2} m_c F_{\mu\nu} \bar{u} \sigma^{\mu\nu} (1 + \gamma_5) c, \quad (4)$$

$$Q_9 = \frac{e^2}{16\pi^2} \bar{u}_L \gamma_\mu c_L \bar{\ell} \gamma^\mu \ell, \quad (5)$$

$$Q_{10} = \frac{e^2}{16\pi^2} \bar{u}_L \gamma_\mu c_L \bar{\ell} \gamma^\mu \gamma_5 \ell. \quad (6)$$

and the corresponding Wilson coefficients  $C_{7,9,10}$ .  $F_{\mu\nu}$  is the electromagnetic field strength, while  $q_L = \frac{1}{2}(1 - \gamma_5)q$  are the left-handed quark fields. Wilson coefficients are taken at the scale  $\mu = m_c$ .

Since we consider exclusive decay modes  $D \rightarrow P\ell^+\ell^-$  we have to employ form factor description of the four-quark operators evaluated between two mesonic states. We use the standard parameterization

$$\langle P(k) | \bar{u} \gamma^\mu (1 - \gamma_5) c | D(p) \rangle = (p + k)^\mu f_+(q^2) + (p - k)^\mu f_-(q^2), \quad (7)$$

$$\langle P(k) | \bar{u} \sigma^{\mu\nu} (1 \pm \gamma_5) c | D(p) \rangle = i s(q^2) \left[ (p + k)^\mu q^\nu - q^\mu (p + k)^\nu \pm i \epsilon^{\mu\nu\alpha\beta} (p + k)_\alpha q_\beta \right], \quad (8)$$

where  $P = \pi^+ (K^+)$  in the case of  $D = D^+ (D_s^+)$ . The momentum transfer  $q = p - k$  is also the momentum of the lepton pair. For the  $f_+$  form factor we use the double pole parameterization of Ref. [36]

$$f_+(q^2) = \frac{f_+(0)}{(1 - x)(1 - ax)}, \quad (9)$$

where  $x = q^2/m_{D^*}^2$ ,  $f_+(0) = 0.617$ , and  $a = 0.579$ . We approximate  $s(q^2)$  by  $f_+(q^2)/m_D$ , which is valid in the limit of heavy  $c$ -quark and zero recoil limit [37]. This relation can be modified as noticed in [38, 39]. However, in our case this modification cannot give significant effects. Finally, we arrive at the short distance amplitude for  $c \rightarrow u\ell^+\ell^-$  decay

$$\mathcal{A}_{\text{SD}} = -i \frac{4\pi\alpha G_F}{\sqrt{2}} V_{cb}^* V_{ub} \left[ \left( \frac{C_7}{2\pi^2} \frac{m_c}{m_D} + \frac{C_9}{16\pi^2} \right) \bar{u}(p_-) \not{p} v(p_+) + \frac{C_{10}}{16\pi^2} \bar{u}(p_-) \not{p} \gamma_5 v(p_+) \right] f_+(q^2). \quad (10)$$

$p$ ,  $p_+$  and  $p_-$  are the momenta of the initial  $D$  meson and the lepton pair in the final state, respectively.

### 1. Standard model

The SM rate is dominated by the photon exchange, where  $c \rightarrow u\gamma$  is a two loop diagram induced by effective weak vertex and a gluon exchange [13, 19, 22, 40]. The effective Wilson coefficient is [13]

$$V_{cb}^* V_{ub} \hat{C}_7^{\text{eff}} = V_{cs}^* V_{us} (0.007 + 0.020i)(1 \pm 0.2). \quad (11)$$

The remaining two Wilson coefficients are subdominant in the SM and we ignore them in further analysis.  $C_9$  is small due to the effects of the renormalization group, while the coefficient  $C_{10}$  is completely negligible in the SM [13].

### 2. Models with extra heavy up vector-like quark singlet

The class of models with an extra up-like quark singlet (EQS) naturally accommodate FCNCs at tree level [13, 24]

$$\mathcal{L}_{NC} = \frac{g}{\cos \theta_W} Z_\mu (J_{W^3}^\mu - \sin^2 \theta_W J_{EM}^\mu). \quad (12)$$

$J_{EM}^\mu$  is the electromagnetic current, while the weak neutral current

$$J_{W^3}^\mu = \frac{1}{2} \bar{U}_L^m \gamma^\mu \Omega U_L^m - \frac{1}{2} \bar{D}_L^m \gamma^\mu D_L^m \quad (13)$$

mixes up-type quarks [29], where  $U^m$  and  $D^m$  are the quark mass eigenstates. The transition matrix to the mass eigenbasis for the up-type quarks is  $4 \times 4$  unitary matrix  $T_L^U$ , which causes tree-level FCNCs in the interaction term  $J_{W^3}^\mu Z_\mu$  in the up sector. The mixing matrix contains only the elements of the last column of matrix  $T_L^U$

$$\Omega = \begin{pmatrix} 1 - |\Theta_u|^2 & -\Theta_u \Theta_c^* & -\Theta_u \Theta_t^* & -\Theta_u \Theta_T^* \\ -\Theta_c \Theta_u^* & 1 - |\Theta_c|^2 & -\Theta_c \Theta_t^* & -\Theta_c \Theta_T^* \\ -\Theta_t \Theta_u^* & -\Theta_t \Theta_c^* & 1 - |\Theta_t|^2 & -\Theta_t \Theta_T^* \\ -\Theta_T \Theta_u^* & -\Theta_T \Theta_c^* & -\Theta_T \Theta_t^* & 1 - |\Theta_T|^2 \end{pmatrix}. \quad (14)$$

The unitarity of the extended CKM matrix then implies that off-diagonal elements of  $\Omega$  are non-zero, e.g.  $\Omega_{uc} \equiv -\Theta_u \Theta_c^* = V_{ud} V_{cd}^* + V_{us} V_{cs}^* + V_{ub} V_{cb}^* \neq 0$ . The low-energy effective description is encoded in Wilson coefficients  $C_9$  and  $C_{10}$ . Relative to the negligible SM values, they are modified by the presence of an extra up-like quark:

$$V_{ub} V_{cb}^* \delta C_9 = \frac{4\pi}{\alpha} \Omega_{uc} (4 \sin^2 \theta_W - 1) \quad (15)$$

$$V_{ub} V_{cb}^* \delta C_{10} = \frac{4\pi}{\alpha} \Omega_{uc}, \quad (16)$$

The element  $\Omega_{uc}$  of the up-type quark mixing matrix is constrained by the measurements of  $D^0 - \bar{D}^0$  mixing [1, 2, 4] and using expression  $\Delta m_D = 2 \times 10^{-7} |\Omega_{uc}|^2 \text{ GeV}$  [29]:

$$\Omega_{uc} < 2.8 \times 10^{-4}. \quad (17)$$

### 3. Minimal supersymmetric SM

The leading contribution to  $c \rightarrow u\ell^+\ell^-$  in general MSSM with conserved  $R$ -parity comes from the gluino exchange diagram via virtual photon and significantly enhances  $c \rightarrow u\ell^+\ell^-$  at small  $m_{\ell\ell}$ . This MSSM enhancement can not be so drastic in hadronic decays, since gauge invariance imposes additional factor of  $m_{\ell\ell}^2$  for  $D \rightarrow P\ell^+\ell^-$  decays, while  $D \rightarrow V\ell^+\ell^-$  has large long distance contribution at small  $m_{\ell\ell}$  just like  $D \rightarrow V\gamma$ .

In the MSSM with broken  $R$ -parity (MSSM $\mathcal{R}$ ), the  $c \rightarrow u\ell^+\ell^-$  process is mediated by the tree-level exchange of down squarks [11]. Integrating them out leads to the effective four-quark interaction

$$\mathcal{L}_{\text{eff}} = \sum_{i,k=1}^3 \frac{\tilde{\lambda}'_{i2k} \tilde{\lambda}'_{i1k}}{2M_{\tilde{d}_R^k}^2} (\bar{u}_L \gamma^\mu c_L) (\bar{\ell}_L \gamma_\mu \ell_L). \quad (18)$$

$\tilde{\lambda}'_{ijk}$  are the CKM-rotated couplings between the  $L$ ,  $Q$  and  $D$  supermultiplets in the superpotential [11]. In our notation (3), the contribution to the Wilson coefficients is [22]

$$V_{cb}^* V_{ub} \delta C_9 = -V_{cb}^* V_{ub} \delta C_{10} = \frac{2 \sin^2 \theta_W}{\alpha^2} \sum_{k=1}^3 \left( \frac{m_W}{M_{\tilde{d}_R^k}} \right)^2 \tilde{\lambda}'_{i2k} \tilde{\lambda}'_{i1k}, \quad (19)$$

where  $i = 1$  (2) contributes to the  $e^+e^-$  ( $\mu^+\mu^-$ ) mode. The  $\tilde{\lambda}'_{12k}$  and  $\tilde{\lambda}'_{11k}$  have been constrained from the charged current universality [22, 41], while the strictest constraint on  $\sum_k \tilde{\lambda}'_{22k} \tilde{\lambda}'_{21k}$  comes [22] from the experimental limit  $\text{Br}(D^+ \rightarrow \pi^+ \mu^+ \mu^-) = 8.8 \times 10^{-6}$  [42]. We shall reanalyze the latter case, where LD physics generates the fair amount of experimental branching ratio. It is sensible to use an approach, where one takes into account the interference between LD and MSSM $\mathcal{R}$  part of the amplitude to constrain the couplings of the MSSM $\mathcal{R}$ .

### B. Long distance contributions in $D \rightarrow P\ell^+\ell^-$

Knowledge of the LD contributions is crucial, if we want to isolate short distance physics in the decays of type  $D \rightarrow P\ell^+\ell^-$ . Following procedure described in [13] we consider long distance contributions by employing the resonant decay modes, in which  $D$  first decays to  $P$  and a virtual

neutral vector meson  $V_0$ , followed by decay of  $V_0 \rightarrow \gamma \rightarrow \ell^+ \ell^-$ . First stage of the decay is controlled by effective weak non-leptonic Lagrangian

$$\mathcal{L}_{\text{eff}}^{\text{LD}} = -\frac{G_F}{\sqrt{2}} \sum_{q=d,s} V_{uq} V_{cq}^* [a_1 \bar{u} \gamma^\mu (1 - \gamma_5) q \bar{q} \gamma_\mu (1 - \gamma_5) c + a_2 \bar{u} \gamma^\mu (1 - \gamma_5) c \bar{q} \gamma_\mu (1 - \gamma_5) q] \quad (20)$$

The effective Wilson coefficients on the scale  $m_c$  are [22]

$$a_1 = 1.26, \quad a_2 = -0.49. \quad (21)$$

The flavour structure of (20) allows  $V_0$  to be either  $\rho$ ,  $\omega$  or  $\phi$ . Since branching ratios of separate stages in the cascade are well measured, we will not work in a particular theoretical model, but will instead try to make the best use of experimental data currently available. Here we follow the lines of Ref. [13]. For a cascade, we write [43]

$$\frac{d\Gamma}{dq^2}(D_s \rightarrow K V_0 \rightarrow K \ell^+ \ell^-) = \frac{1}{\pi} \Gamma_{D_s \rightarrow K V_0}(q^2) \frac{\sqrt{q^2}}{(m_{V_0}^2 - q^2)^2 + m_{V_0}^2 \Gamma_{V_0}^2} \Gamma_{V_0 \rightarrow \ell^+ \ell^-}(q^2). \quad (22)$$

Here  $\Gamma_{D_s \rightarrow K V_0}(q^2)$  and  $\Gamma_{V_0 \rightarrow \ell^+ \ell^-}$  denote decay rates if  $V_0$  had a mass  $\sqrt{q^2}$  and these rates are known experimentally only at  $\sqrt{q^2} = m_{V_0}$ . Since the resonances  $V_0 = \rho, \omega, \phi$  are relatively narrow ( $\Gamma_{V_0} \ll m_{V_0}$ ), the following relation approximately holds

$$\text{Br}[D \rightarrow P V_0 \rightarrow P \ell^- \ell^+] = \text{Br}[D \rightarrow P V_0] \times \text{Br}[V_0 \rightarrow \ell^- \ell^+]. \quad (23)$$

The phenomenological amplitude ansatz that reproduces the above behaviour is then [13]

$$\mathcal{A}^{\text{LD}}[D_s(p) \rightarrow K(p-q) V_0(q) \rightarrow K(p-q) \ell^-(p_-) \ell^+(p_+)] = e^{i\phi_{V_0}} \frac{a_{V_0}}{q^2 - m_{V_0}^2 + im_{V_0} \Gamma_{V_0}} \bar{u}(p_-) \not{p} v(p_+). \quad (24)$$

The only assumption we made here is that coefficient  $a_{V_0}$  is independent of  $q^2$ . We included the phase  $\phi_{V_0}$  explicitly, so that  $a_{V_0}$  is real and positive number.

### 1. $D^+ \rightarrow \pi^+ \ell^+ \ell^-$

For the right side of Eq. (23) we use experimental data (Table II), and use ansatz (24) to extract unknown parameters  $a_{V_0}$ :  $a_\rho = 2.94 \times 10^{-9}$ ,  $a_\phi = 4.31 \times 10^{-9}$ . Decay mode  $D^+ \rightarrow \pi^+ \omega$  has not

mode	$D^+ \rightarrow \pi^+ \rho$	$D^+ \rightarrow \pi^+ \omega$	$D^+ \rightarrow \pi^+ \phi$
$\text{Br} \times 10^3$	$1.07 \pm 0.11$	$< 0.34$	$6.50 \pm 0.70$

Table II: Branching ratios of decays of  $D^+$  meson to the intermediate resonant states [44].



been measured yet, but we can relate  $a_\omega$  and its phase to the well-measured contribution of the  $\rho$  resonance assuming vector meson dominance as in [13]. Relative phases and magnitudes of the resonances are extracted by considering the decay mechanism, controlled by the weak Lagrangian (20) and electromagnetic coupling of  $V_0$  to photon. Then flavour structure of the resonances determines relative sizes and phases of resonant amplitudes. Detailed analysis has already been done in Ref. [13]. The relative phases of  $\rho$  and  $\omega$  contributions are found to be opposite in sign, while for the ratio of their magnitudes it was found that  $a_\omega/a_\rho = 1/3$ . Also the phases of  $\rho$  and  $\phi$  are opposite. Thus the final LD amplitude (up to the phase) becomes

$$i\mathcal{M}^{\text{LD}} = \left[ a_\rho \left( \frac{1}{q^2 - m_\rho^2 + im_\rho\Gamma_\rho} - \frac{1}{3} \frac{1}{q^2 - m_\omega^2 + im_\omega\Gamma_\omega} \right) - \frac{a_\phi}{q^2 - m_\phi^2 + im_\phi\Gamma_\phi} \right] \bar{u}(p_-) \not{p} v(p_+). \quad (25)$$

## 2. $D_s^+ \rightarrow K^+ \ell^- \ell^+$

Experimental data is not as rich as in the case of non-strange charmed meson decays (Table III). Contributions of  $\rho$  and  $\omega$  are related like in the case of  $D^+$  meson, namely  $a_\omega/a_\rho = 1/3$  with

mode	$D_s^+ \rightarrow K^+ \rho$	$D_s^+ \rightarrow K^+ \omega$	$D_s^+ \rightarrow K^+ \phi$
$\text{Br} \times 10^3$	$2.60 \pm .70$	—	$< 0.50$

Table III: Branching ratios of  $D_s^+$  meson to the intermediate resonant state [44].

opposite relative phase between them. In the same way as for the non-strange decays, we determine  $a_\rho = 6.97 \times 10^{-9}$ . However, the contribution of the  $\phi$  resonance is only limited from above by experimental data and we have to rely on a theoretical model. Consequently, the total LD amplitude is a sum of two terms:

$$\mathcal{A}^{\text{LD}} = a_\rho \left( \frac{1}{q^2 - m_\rho^2 + im_\rho\Gamma_\rho} - \frac{1}{3} \frac{1}{q^2 - m_\omega^2 + im_\omega\Gamma_\omega} \right) \bar{u}(p_-) \not{p} v(p_+) + \mathcal{A}_\phi^{\text{LD}}. \quad (26)$$

We calculate the  $\phi$  part of (26) using the vector meson dominance (VMD) assumption, where the intermediate  $\phi$  contributes by decaying into a virtual photon, which further decays to the lepton pair. Both  $a_1$  and  $a_2$  parts of the non-leptonic Lagrangian (20) can generate the flavour quantum numbers of  $\phi$  and  $K^+$ . The  $a_1$  part connects initial  $D_s^+$  state to  $\phi$  through a charged current  $(\bar{s}c)_{V-A}$ , while the  $(\bar{u}s)_{V-A}$  creates the  $K^+$  out of vacuum. Neutral currents (the  $a_2$  part) do the opposite:  $D_s^+ \rightarrow K^+$  and  $0 \rightarrow \phi$ . Utilizing the Feynman rules of (20), and the VMD hypothesis

we arrive at the  $\phi$  contribution to the LD amplitude

$$\mathcal{A}_\phi^{\text{LD}} = i \frac{4\pi\sqrt{2}}{3} G_F V_{us} V_{cs}^* \alpha \frac{g_\phi}{q^2(q^2 - m_\phi^2 + im_\phi\Gamma_\phi)} [a_1 m_\phi f_K A_0(m_K^2) + a_2 g_\phi f_+(q^2)] \bar{u}(p_-) \not{p}_v(p_+). \quad (27)$$

However, the phase of  $\mathcal{A}_\phi^{\text{LD}}$  relative to the rest of the amplitude (26) remains to be free. The  $P \rightarrow V$  transition  $D_s^+ \rightarrow \phi$  is described by the form factor  $A_0$ , which we take from [45]. In our calculations we consider gauge invariant amplitude for  $D_s^+ \rightarrow K^+ \ell^+ \ell^-$  in which  $1/q^2$  dependence is cancelled.

#### IV. RESULTS

Using the approach, described in Sec. 3, we analyze impact of short distance physics on long-distance resonant background. Since the SD contribution of SM is completely overshadowed by LD, we will only consider the EQS and MSSM $\mathcal{R}$  models of new physics, to see if the experimental searches for them are still viable in  $D \rightarrow X \ell^+ \ell^-$  decays. Current constraints on EQS model coming from the  $D^0 - \bar{D}^0$  mixing already indicate the dominance of LD contributions in the total decay rate. On the other hand, the contribution of MSSM $\mathcal{R}$  is not as constrained and one should still see the deviations from the LD contribution away from the resonant region of the phase space.

We shall analyze the dilepton squared mass ( $q^2 = m_{\ell\ell}^2$ ) distribution of the branching ratios. We fix free phases in the amplitude in a way which maximizes branching ratio for the considered decay mode.

##### A. $D^+ \rightarrow \pi^+ \ell^+ \ell^-$

mode	LD	extra heavy quark	LD+extra heavy quark	MSSM $\mathcal{R}$	LD+MSSM $\mathcal{R}$
$D^+ \rightarrow \pi^+ e^+ e^-$	$2.0 \times 10^{-6}$	$1.3 \times 10^{-9}$	$2.0 \times 10^{-6}$	$2.1 \times 10^{-7}$	$2.3 \times 10^{-6}$
$D^+ \rightarrow \pi^+ \mu^+ \mu^-$	$2.0 \times 10^{-6}$	$1.6 \times 10^{-9}$	$2.0 \times 10^{-6}$	$6.5 \times 10^{-6}$	$8.8 \times 10^{-6}$

Table IV: Total branching fractions of the  $D^+ \rightarrow \pi^+ \ell^+ \ell^-$  modes. In the first column (LD) are only long-distance BRs. The remaining four columns give maximal contributions of the SD physics models alone and also combined contributions of the SD and LD physics.

Branching fractions are listed in Table IV. Clearly, the EQS model contribution is too small to be observed (Fig. 1).

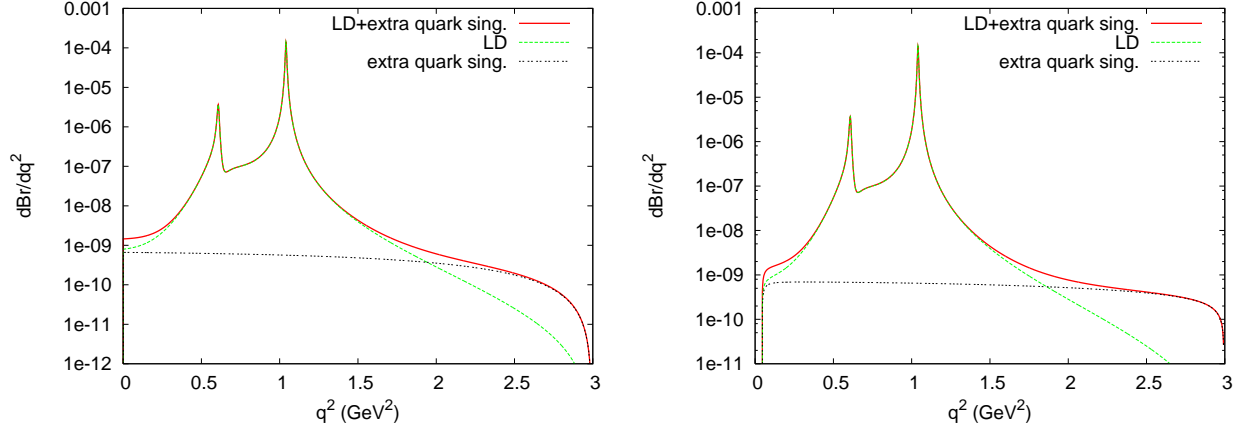


Figure 1: Distributions of the maximal branching ratios in the model with extra quark singlet for the decay modes  $D^+ \rightarrow \pi^+ e^+ e^-$  (left) and  $D^+ \rightarrow \pi^+ \mu^+ \mu^-$  (right). Full line represents the combined LD and SD contributions.

On the other hand, the  $\text{MSSM}\overline{R}$  gives a slight increase to the mode with electrons. Deviation from the LD amplitude is pronounced in the region without resonances, where  $m_{\ell\ell} < m_\rho$  or  $m_{\ell\ell} > m_\phi$  (Fig. 2, left). However, the most promising mode is the channel with muons. The long-

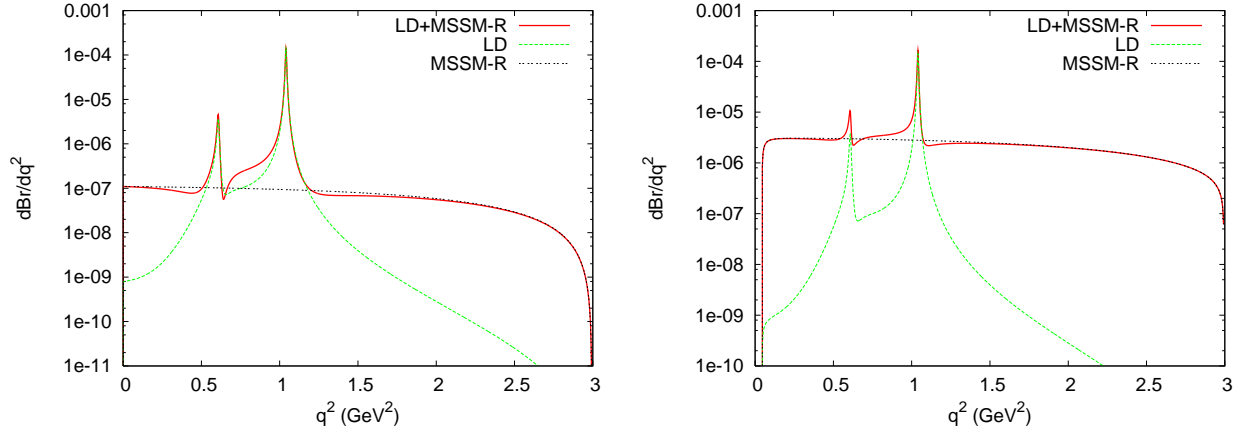


Figure 2: Distributions of the maximal branching ratios in the  $\text{MSSM}\overline{R}$  model for the decay modes  $D^+ \rightarrow \pi^+ e^+ e^-$  (left) and  $D^+ \rightarrow \pi^+ \mu^+ \mu^-$  (right). Full line represents the combined LD and SD contributions, and it corresponds to the experimental upper bound  $BR(D^+ \rightarrow \pi^+ \mu^+ \mu^-) = 8.8 \times 10^{-6}$  on the right plot.

distance contribution ( $2.2 \times 10^{-6}$ ) is a fair share of the experimental upper bound [42] ( $8.8 \times 10^{-6}$ ) and should be taken into account together with the short distance part, when one is constraining the Wilson coefficients. The difference is not big, i.e. when we drop the LD part we get for the

bound  $|V_{cb}^* V_{ub} C_{9,10}^\mu| < 27$ , while the analysis with LD part included gives

$$|V_{cb}^* V_{ub} C_{9,10}^\mu| < 23. \quad (28)$$

The latter bound is a maximum with respect to the free relative phase between the LD and MSSM $\mathcal{R}$  parts of the amplitude. Although the inclusion of the LD term does not make substantial difference, it will grow rapidly as experimental bound is approaching  $2.2 \times 10^{-6}$ . All the branching ratios concerning MSSM $\mathcal{R}$  and muons in the final state (Table IV,V) and their kinematical distributions (Fig. 2,4) use the bound (28).

### B. $D_s^+ \rightarrow K^+ \ell^+ \ell^-$

The branching ratios contributions are summarized in Table V. Again, the EQS model has negligible effect (Fig. 3). MSSM $\mathcal{R}$  has a notable effect, especially in the  $\mu^+ \mu^-$  mode, where it increases branching ratio by an order of magnitude (Fig. 4). In this case, the MSSM $\mathcal{R}$  overshadows the LD contribution throughout the phase space, except in the close vicinity of the LD resonant peaks.

mode	LD	EQS	LD+EQS	MSSM $\mathcal{R}$	LD+MSSM $\mathcal{R}$
$D_s^+ \rightarrow K^+ e^+ e^-$	$6.0 \times 10^{-7}$	$5.4 \times 10^{-10}$	$6.0 \times 10^{-7}$	$9 \times 10^{-8}$	$7.6 \times 10^{-7}$
$D_s^+ \rightarrow K^+ \mu^+ \mu^-$	$6.0 \times 10^{-7}$	$6.2 \times 10^{-10}$	$6.0 \times 10^{-7}$	$2.6 \times 10^{-6}$	$3.6 \times 10^{-6}$

Table V: Total branching fractions of the  $D_s^+ \rightarrow K^+ \ell^+ \ell^-$  modes. In the first column (LD) are only long-distance BRs. The remaining four columns give maximal contributions of the SD physics models alone and also combined contributions of the SD and LD physics.

## V. SUMMARY

Recently observed  $D^0 - \bar{D}^0$  mass difference constrains the value of tree-level flavor changing neutral coupling  $c \rightarrow uZ$ , which is present in the models with an additional singlet up-like quark. We have studied the impact of this coupling on rare  $D^+ \rightarrow \pi^+ \ell^+ \ell^-$  and  $D_s^+ \rightarrow K^+ \ell^+ \ell^-$  decays, where its effects are accompanied by the long distance contributions. We have determined long-distance contributions in  $D_s^+ \rightarrow K^+ \ell^+ \ell^-$  following the same phenomenologically inspired model as it has been done previously in the case of  $D^+ \rightarrow \pi^+ \ell^+ \ell^-$ . We find that the effect of new extra singlet up-like quark is too small to be seen in dilepton mass distributions for both decay modes. In our previous study we have considered forward-backward asymmetry in the  $D^0 \rightarrow \rho^0 \ell^+ \ell^-$  and

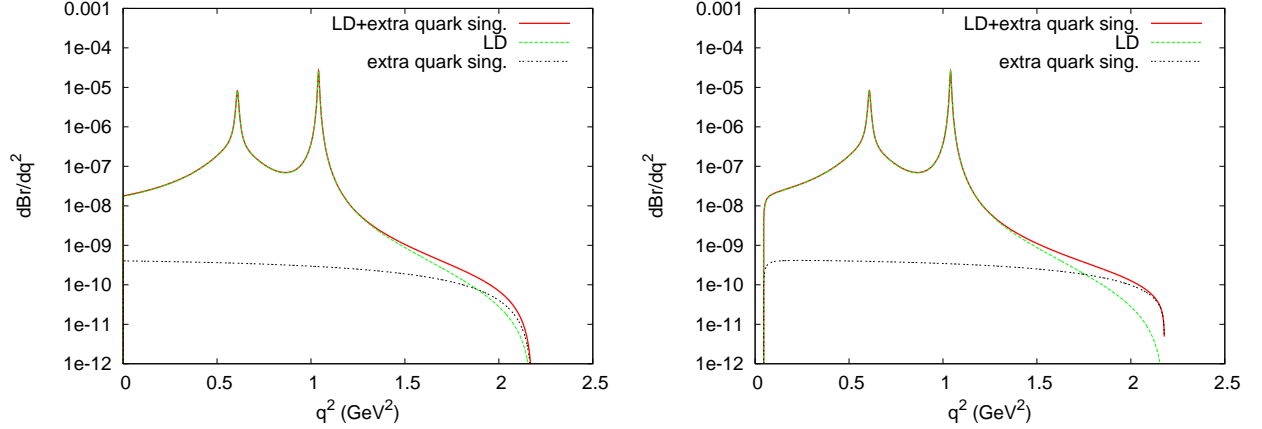


Figure 3: Distributions of the maximal branching ratios in the model with extra quark singlet for the decay modes  $D_s^+ \rightarrow K^+ e^+ e^-$  (left) and  $D_s^+ \rightarrow K^+ \mu^+ \mu^-$  (right). Full line represents the combined LD and SD contributions.

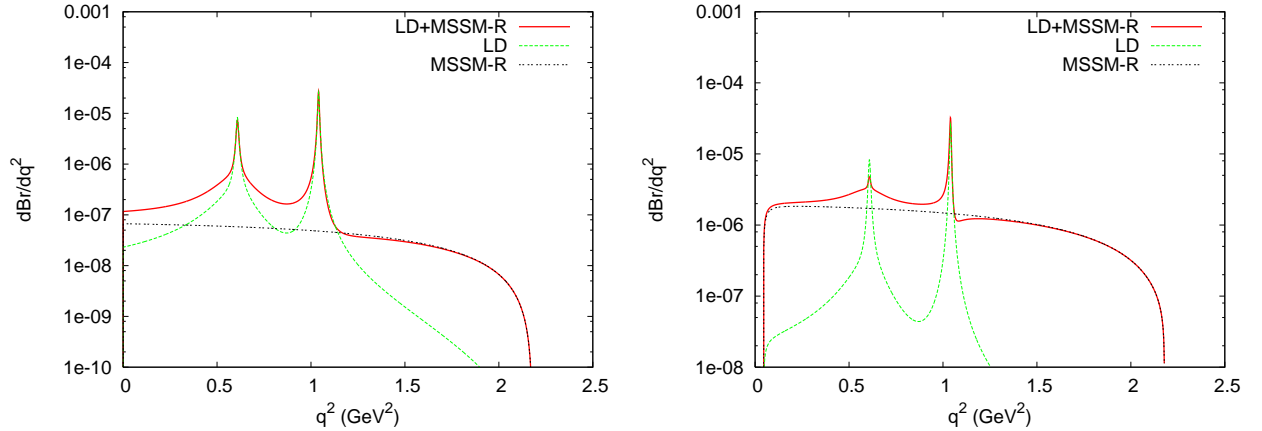


Figure 4: Distributions of the maximal branching ratios in the MSSM-R model for the decay modes  $D_s^+ \rightarrow K^+ e^+ e^-$  (left) and  $D_s^+ \rightarrow K^+ \mu^+ \mu^-$  (right). Full line represents combined LD and SD contributions.

found very small effect. New constraint reduces that asymmetry even more, making it insignificant for the experimental searches.

Present constraints on mass insertions in MSSM with conserved  $R$ -parity still allow for increase of  $c \rightarrow u\gamma$  rate by one order of magnitude. For the same reason MSSM could significantly increase  $c \rightarrow u\ell^+\ell^-$  rate at small  $m_{\ell\ell}$ . However, this MSSM enhancement is not drastic in  $D$  decays, since  $D \rightarrow V\gamma$  and  $D \rightarrow V\ell^+\ell^-$  have large long distance contributions for small  $m_{\ell\ell}$ , while  $D \rightarrow P\ell^+\ell^-$  rate is multiplied by factor of  $m_{\ell\ell}^2$  due to gauge invariance.

The remaining possibility to search for new physics in rare  $D$  decays is offered by the MSSM models which contain  $R$ -parity violating terms. We reinvestigate bounds on the combinations of

these parameters in  $D^+ \rightarrow \pi^+ \ell^+ \ell^-$  by including the long-distance effects. Using current upper bound on the rate for  $D^+ \rightarrow \pi^+ \ell^+ \ell^-$  we derive new bound:

$$\sum_{k=1}^3 \left( \frac{100 \text{ GeV}}{M_{\tilde{d}_R^k}} \right)^2 \tilde{\lambda}'_{21k} \tilde{\lambda}'_{22k} < 0.0041 \quad (29)$$

Since at Tevatron there are plans to investigate  $D_s^+ \rightarrow K^+ \ell^+ \ell^-$  decay we use upper bound (29) and calculate dilepton invariant mass distribution. This bound still gives small increase of the dilepton invariant mass distribution for the larger invariant dilepton mass, making it attractive for the planned experimental studies.

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